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MAGNETO-FLUID-MECHANIC LAMINAR NATURAL CONVECTION—AN EXPERIMENT

D. D. PAPAILIOU and P. S. LYKOUDIS
Purdue University, Lafayette, Indiana, U.S.A.

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Abstract—This work describes an experiment in which the effect of a magnetic field on a laminar natural convection of electrically conducting fluid is studied. Mercury was the working fluid. The geometry was the one of a vertical isothermal hot plate with a magnetic field acting perpendicularly. The magnetic field imposed externally was designed as suggested by the theoretical investigation for the existence of a similarity solution. This suggestion is that the magnetic flux should vary as $x^{-\frac{1}{4}}$ along the vertical direction, x.

Temperature profiles under the above conditions have been measured for different magnetic field strengths. The existence of similarity solutions was fully established by the experiment.

NOMENCLATURE

B, magnetic flux;

C, quantity defined by

$$C = \frac{g(T_w - T_\infty)}{4v^2 T_\infty};$$

c_p, specific heat;

g, gravitational constant;

Gr, Grashof number,

$$= \frac{gx^3(T_w - T_\infty)}{v^2 T_\infty};$$

k, thermal conductivity;

Pr, Prandl number, $= c_p\mu/k$;

q, heat flux;

T, temperature;

x, y, Cartesian coordinates along and normal to the plate;

Z, magnetic interaction parameter, $= \sigma B^2 x^{\frac{1}{2}}/\rho v C^2$.

Greek symbols

η, similarity parameter, $= Cy/\sqrt[4]{x}$;

θ, nondimensional temperature, $= (T - T_\infty)/(T_w - T_\infty)$;

μ, viscosity;

v, kinematic viscosity;

ρ, mass density;

σ, electrical conductivity.

Subscripts

w, refers to the wall;

∞, refers to the free stream;

x, y, components in the x and y directions.

INTRODUCTION

THIS work presents the results of an experimental investigation of the effect of a magnetic field on the laminar natural convection of an electrically conducting fluid. The case examined in this experiment is that of a vertical hot plate of uniform temperature with mercury as conducting fluid, in the presence of a transverse magnetic field (Fig. 1).

The above case was theoretically investigated by Lykoudis [1] and independently by Gupta [2]. The theoretical assumptions made in solving the problem are the following:

(a) The magnetic field is constant in the direction perpendicular to the wall, throughout the thickness of the boundary layer.

(b) There is no appreciable distortion of the magnetic lines due to the velocity field. This implies a small magnetic Reynolds number, which is the case for experiments with liquid metals.

(c) The coefficient of electrical conductivity is a scalar and remains constant everywhere.

(d) The electric field, calculated in the same frame of reference in which velocity is measured, is zero.

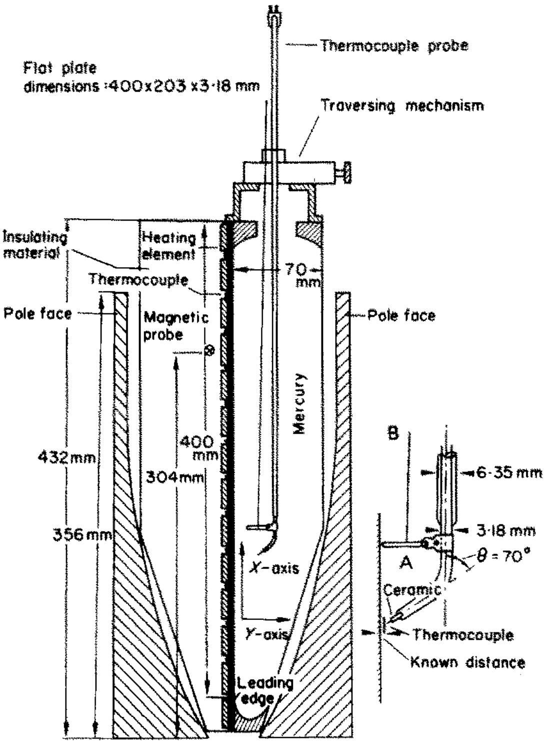

FIG. 1. Experimental apparatus.

Under the above assumptions it has been found that when the momentum equation is cast in its similarity form the ponderomotive term is multiplied by the nondimensional number $Z = \sigma B^2 x^{\frac{1}{2}}/\rho \nu C^2$. This number can be physically identified as the ratio of the ponderomotive force divided by the square root of the product of the buoyant and inertial forces.

It is evident that, when the magnetic field varies as $x^{-\frac{1}{4}}$, Z becomes independent of the distance from the leading edge of the plate; this is the necessary and sufficient condition for the existence of a similarity solution. For this case, the heat flux at the wall of the hot plate is given as follows:

$$q_w = \frac{-KC(T_w - T_\infty)}{\sqrt[4]{x}} \cdot \theta'_w. \tag{1}$$

In the above, θ'_w is the nondimensional temperature gradient at the wall.

$$\theta'_w = \left(\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right)_{\eta=0} \tag{2}$$

The following approximate expression for θ'_w was obtained in [1]:

$$\theta'_w = \left\{ \frac{\dfrac{-7Z}{8Pr} + \left[\left(\dfrac{7Z}{8Pr}\right)^2 + 2\left(\dfrac{21}{4Pr} + \dfrac{7}{8Pr^2}\right)\right]^{\frac{1}{2}}}{2\left(\dfrac{21}{4Pr} + \dfrac{7}{8Pr^2}\right)} \right\}^{\frac{1}{2}} \tag{3}$$

An exact solution obtained with the use of an analogue computer is also reported in the same reference.

The approximate expression obtained for θ'_w in [2] is:

$$-\theta'_w = \frac{1}{\sqrt{3}}\left(\frac{Pr}{\frac{20}{21} + Pr}\right)^{\frac{1}{2}} \left\{\left[\frac{Z}{4} + \frac{12}{5}Pr + \frac{16}{7}\right]^{\frac{1}{2}} - \frac{Z}{2}\right\}^{\frac{1}{2}} \tag{4}$$

The experiment reported here was designed in such a way as to satisfy all of the above theoretical conditions.

EXPERIMENTAL APPARATUS

The construction of the experimental cell, based on past experience such as described in [3], was of stainless steel and is shown in Fig. 1. One of the vertical walls of the cell was used as a hot plate, the other wall had the shape shown in Fig. 1, to fit with the shape of the pole faces of the electromagnet. The whole cell was insulated on all sides except for the nonheated vertical wall. Fourteen heating elements were uniformly spaced on the external face of the hot plate. Eighteen thermocouples were spaced along both directions of the plate so that an accurate picture of the temperature distribution on the plate was obtained.

In order to have a uniform temperature along the heated plate, the power distribution along the wall should be proportional to the minus inverse fourth power of the distance from the lower edge of the plate. For this purpose, each of the heating elements was properly connected to either a Variac or a resistor system, to control the power delivered to the plate. A system which consisted of a wattmeter and a combination of a voltmeter and ammeter, was used through a multipole switch to measure the power for each element.

The temperature distribution in the thermal boundary layer was measured by a traversing probe consisting of an iron–constantan thermocouple supported by a 605 mm stainless steel tube.

Two thermocouples of 0·18 mm and 0·27 mm head diameters enameled with a thin coat of insulation were used. Their total diameter did not exceed 0·3 mm in either case.

The probe was driven by a traversing mechanism capable of measuring distances of 0·025 mm. The driving mechanism was placed on the top of the cell, as shown in Fig. 1. The probe was able to move in the vertical direction to measure the temperature profile at different heights. The problem of measuring the exact location of the thermocouple with respect to the wall was solved by using the mechanical system shown in Fig. 1. An arm made from stainless steel was pivoted at the stem of the probe. It was able to rotate around the point A by means of the wire B so that when the edge of the arm touched the wall the location of the thermocouple head relative to the wall was known. The system was calibrated outside the cell; it was found that the location of the head could be determined with an accuracy of 0·025 mm.

The problem of the design of a magnetic field which changes along the vertical direction according to the relation $B \sim x^{-\frac{1}{4}}$ was reported by MacGregor and Lykoudis [3]. In the present experiment the same pole faces were used as in the work reported above. The overall dimensions of the poles were 356 × 457 mm. They were positioned vertically between the pole faces of the large electromagnet of the Magneto-Fluid-Mechanics Laboratory which are 1270 mm in length, 305-mm wide and have a gap of 203 mm. Measurements with the help of a gaussmeter showed that the $x^{-\frac{1}{4}}$ law along the vertical direction was valid everywhere with the exception of 90 mm from the lower and upper end of the poles because of fringe effects. On the other hand, one fundamental theoretical requirement mentioned in the introduction, was that the intensity of the flux in the horizontal direction across the poles must be constant over the thickness of the boundary layer. Even though the boundary layer thickens with increasing field, it was found that this condition was observed everywhere in all the experiments to a high degree of accuracy.

MEASUREMENTS

The time required for steady-state conditions to be established in the cell was about 14 h, while the steady conditions were established in about 1 h when the intensity of the magnetic field was changed.

The temperature distribution along the plate, measured by the thermocouples located on the back of the plate, indicated a variation on temperature of 2–3 per cent for all cases, except at the ends of the plate. All the thermocouples were taken through a multipole-switch and an ice–water junction to a Leeds and Northrop K-2 Potentiometer.

In order to determine the temperature at the wall, denser measurements very close to the wall were taken. The wall temperature was determined then, by extrapolation of the temperature profile. In this manner, the distortion of the temperature profile, due to the presence of the probe very near the wall, was determined.

Two sets of measurements were taken, one with and the other without the presence of the magnetic field. For the nonmagnetic case, the temperature profiles were measured at different positions along the plate for two different values of total applied power. The levels of the applied power were limited to a rather narrow range in order to prevent the development of turbulent flow ($Gr \times Pr > 10^9$) or a very thick thermal boundary layer.

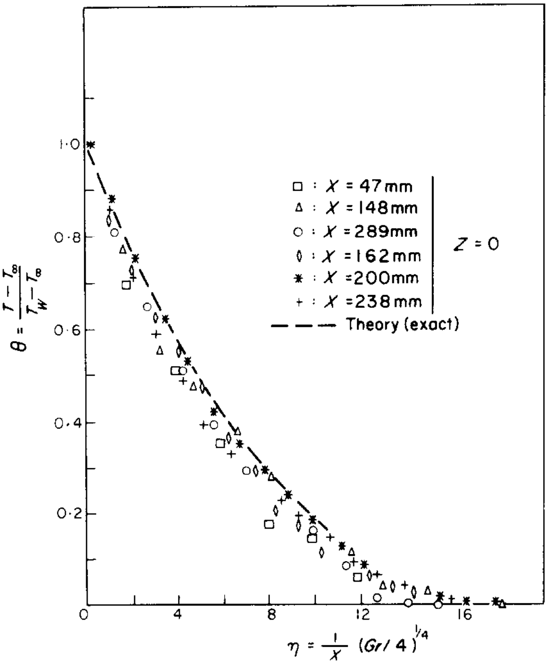

FIG. 2. Temperature profiles plotted in similarity coordinates for zero magnetic field.

The two selected values for the applied power were 27 and 42 W. The results of these measurements are given in Fig. 2. In all measurements with a magnetic field present the power supplied was 42 W.

Figure 3 shows the temperature distribution between the two walls of the cell at a distance $x = 250$ mm from the leading edge of the plate. It can be seen, for increasing magnetic field, how from a free convection flow an almost purely conductive pattern is reached at a locally measured, magnetic field of 4300 G.

Temperature profiles taken for magnetic fields corresponding to mean interaction parameters $\bar{Z} = 0{\cdot}995$ and $\bar{Z} = 2{\cdot}4$, are presented in similarity coordinates in Figs. 4 and 5. For higher magnetic fields the thermal boundary layers of the two vertical walls were mixed, and the use of similarity coordinates was not possible.

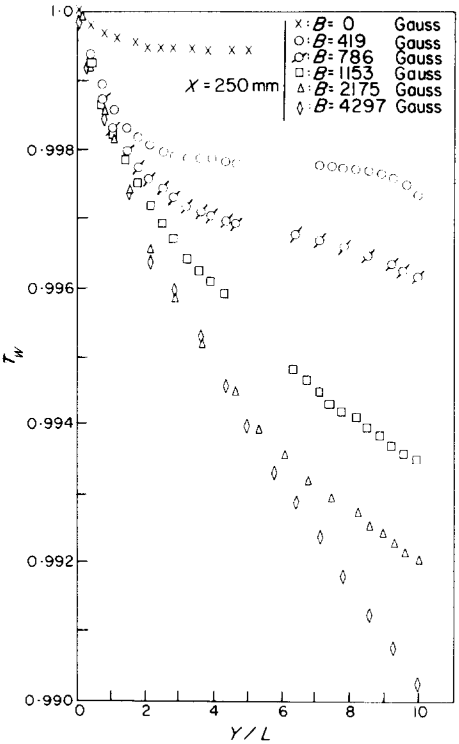

FIG. 3. Temperature distribution between the walls of the cell, taken at $x = 250$ mm for different magnetic field intensities.

SOURCES OF EXPERIMENTAL ERRORS

Possible sources of errors in these measurements, beyond the usual errors due to the accuracy of the instrumentation used, are the following:

(a) The size of the thermocouple head.

(b) Possible inaccuracy in measuring the distance of the head of the thermocouple from the wall.

(c) Variations in temperature distribution over very long periods due to change of the external conditions of temperature inside the laboratory and also in fluctuations in the applied power.

(d) Errors in measuring the wall temperature.

To eliminate errors due to (a) two probes of different size were used. No difference in the measurements was detected. This should be expected since the temperature gradient is so small, even near the wall, that in the volume occupied by the probe the temperature is practically constant.

Concerning errors due to (b), it was previously mentioned that the mechanical system used for the initial location of the probe (at $y = 0$), introduces a maximum systematic error of 0·025 mm. On the other hand, the vernier of the traversing mechanism, moving the probe in the y-direction, measures with an accuracy of 0·025 mm. These numbers are negligible compared to the smallest thickness of the boundary layers measured (about 25 mm).

The most serious source for error was the slow variation in the temperature distribution mentioned in (c) above. It was found, however, that during the time needed for the measurement of a single temperature profile from the wall to the edge of the boundary layer (about 20 min) the variation was negligible. On the other hand, in order to measure the temperature distribution along the distance between the two walls shown in Fig. 3, it was necessary to stop the measurements at about the middle point of the gap, reverse the probe position, wait for steady state to be reestablished and start measuring from the cool wall towards the middle point. Generally, it was found that the temperature at the same middle point had changed during the time interval (about 2 h) between the two measurements, the change was found to be between 0·02 and 0·06 degC for the different sets of experiments. In order to allow for this discrepancy, the temperature distribution measured from the two sides was matched at the middle point. Assuming that the temperatures measured at the side of the cool wall varied linearly with time, an appropriate correction was applied at the points where these measurements were taken.

RESULTS AND CONCLUSIONS

The profiles presented in Fig. 3 in similarity

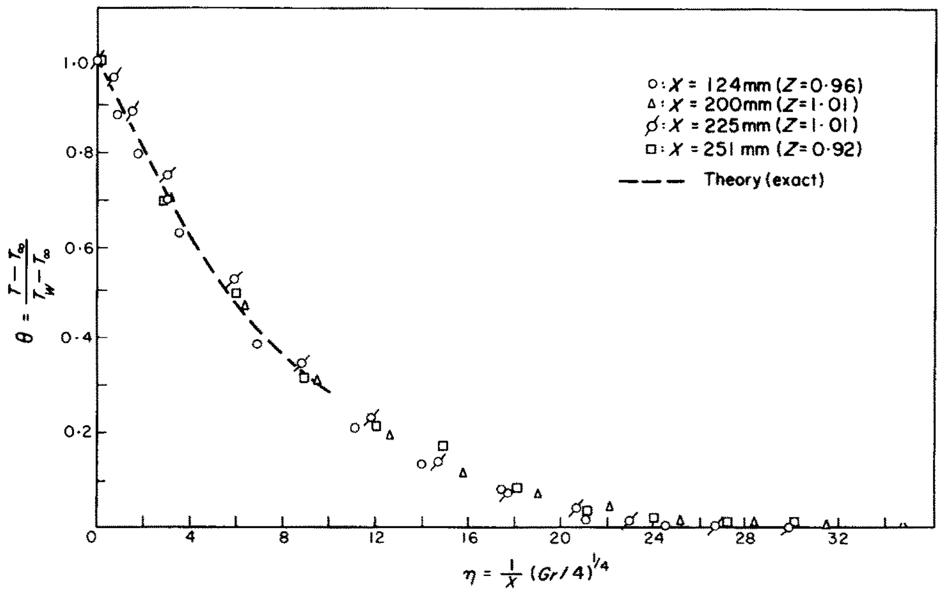

FIG. 4. Temperature profiles plotted in similarity coordinates for a mean magnetic interaction parameter $\bar{Z} \simeq 1{\cdot}0$.

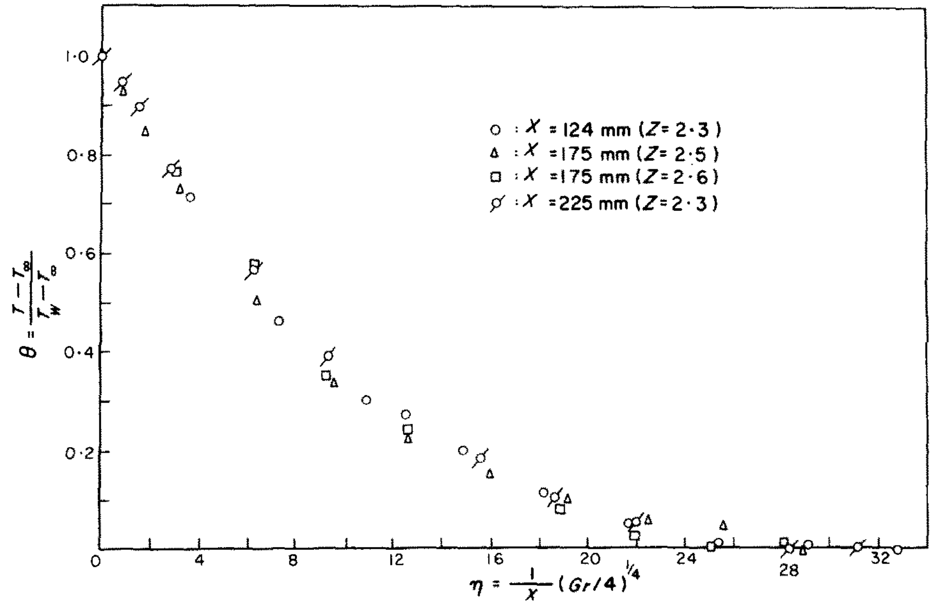

FIG. 5. Temperature profiles plotted in similarity coordinates for a mean magnetic interaction parameter $\bar{Z} \simeq 2·4$.

coordinates for the nonmagnetic case ($Z = 0$) prove experimentally the existence of similarity. It can also be seen that the experimental values form a mean profile lying about 10 per cent below the theoretically predicted one [3]. To the knowledge of the authors these are the first measurements in liquid metals reported in the literature.

Figures 4 and 5 prove also that similarity exists when a magnetic field is present according to the $x^{-\frac{1}{4}}$ law. In Fig. 4 the mean interaction parameter $\bar{Z}$, was equal to 0·995, a case for which an exact theoretical solution is available in [3].

It can be seen that the agreement of theory and experiment is good. Figure 5 shows the experimental results for $\bar{Z} = 2·4$. No theoretical profile higher than $Z = 1$ is available. However, here again the data indicate the validity of similarity since they cluster in one curve.

A comparison with the experimental results of Emery [4] is not possible, since his experiment was conducted with a constant and uniform magnetic field. In our experiment, however, it was found that at locally measured magnetic fields higher than 4000 G, convection still exists, as can be seen in Fig. 3; its influence on heat transfer in the cell is probably minor.

Finally the values of θ_w' computed from the

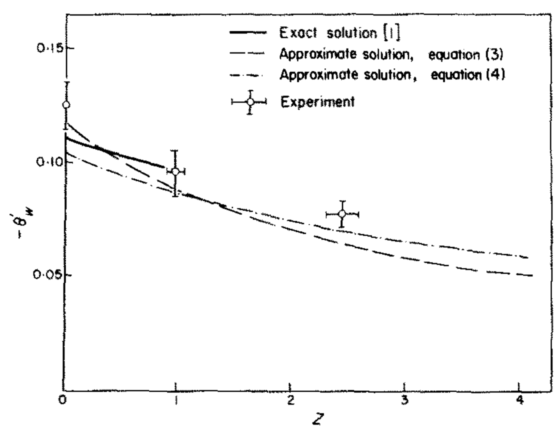

FIG. 6. Variation of θ_w' with $\bar{Z}$.

relevant measured quantities for different Z's, are presented in Fig. 6.

Each cross represents one of the groups of similarity profiles shown in Figs. 2, 4 and 5. It is seen that the experimental data agree well with the theory.

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Résumé—Ce travail décrit une expérience dans laquelle on étudie l'effet d'un champ magnétique sur la convection naturelle laminaire d'un fluide conducteur de l'électricité. Le fluide utilisé était le mercure et la configuration était constituée par une plaque chaude verticale et isotherme avec un champ magnétique agissant perpendiculairement. Le champ magnétique était imposé extérieurement de la façon suggérée par une étude théorique de l'existence d'une solution en similitude, c'est-∂-dire que le flux magnétique devrait varier comme $x^{-\frac{1}{4}}$, x étant dirigé verticalement.

Les profils de température correspondant aux conditions cidessus ont été mesurés pour différentes intensités de champ magnétique. L'existence de solutions en similitude a été entièrement établie par l'expérience.

Zusammenfassung—Es wird ein Experiment beschrieben, in dem der Einfluss eines Magnetfeldes auf die laminare freie Konvektion einer elektrisch leitenden Flüssigkeit untersucht wird. Quecksilber diente als Arbeitsmedium. Das Magnetfeld wirkte senkrecht zu einer beheizten, isothermen, senkrechten Platte. Das von aussen aufgebrachte Magnetfeld entsprach einer theoretischen Untersuchung über die Existenz einer Ähnlichkeitslösung. Danach müsste sich der Magnetfluss in der senktechten Richtung x mit $x^{-\frac{1}{4}}$ ändern.

Unter den angegebenen Bedingungen wurden Temperaturprofile für vershiedene magnetische Feldstärken untersucht. Die Existenz von Ähnlichkeitslösungen wurde durch das Experiment voll bestätigt.